ANALYSIS OF ALIGNED MAGNETIC FIELD IN CASSON FLUID PAST A SHRINKING SHEET

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Abstract-This article deliberates the impact of aligned MHD flow in a Casson fluid on heat and flow transfer. The influence of various fluid parameter like Casson fluid parameter, magnetic parameter, Prandtl number, aligned angle parameter. The governing partial differential equations produced the heat and flow transportaion are moulded to ordinary differential equations by considering similarity transformations. The scientific results of the differential equations have been figure out by the Runge-Kutta Fehlberg rule with the help of shooting technique. The diversity in behavior of emerging parameters characterized graphically and their results have been discussed through table.
Keywords: Casson fluid, Aligned MHD, Shrinking sheet, Boundary layer.

1. INTRODUCTION

Stretching sheet problems under the influence of boundary layer flow becomes a part of attraction for many researchers due to its mathematical simplicity. On the other hand, there is vital application of stretching sheet in industries as well as in engineering areas like metal spinning, polymer processing and drawing of plastic film. [1]–[4] shows the effect of boundary layer flow over stretching sheet.

During research stretching sheet becomes part of study from many years ago, after that shrinking sheet takes a concern form many researchers. Literature for flow towards shrinking sheet is limited as comparative to stretching sheet. Still after this limitation there are many applications of flow over shrinking sheet such that polymer sheets, manufacturing of filaments, glass-fiber and paper production whereas wide area of application chemical engineering and manufacturing industries also considered. Miklavcic and Wang [5] investigates characteristics of flow over shrinking sheet with suction effects while Fang and Zhong [6] discussed the effect of boundary layer flow with arbitrary velocity flow. Khan et al. [7] analyzed the heat transfer effects on horizontal stretching/shrinking sheet.

Stagnation flow exits due to its importance in boundary layer flow. Problem is observed in case of stretching and shrinking balloon. It is noticed that solutions does not exists due to boundary layer flow, so after adding the effect of stagnation flow on boundary layer make possibility of similarity solution that gives exact results. Wang [8] considered the effects of stagnation flow over shrinking sheet whereas [9]–[12] all concern with heat transfer analysis of boundary layer flow with stagnation point over stretching/shrinking sheet. Mahapatra and Nandy [13] taken unsteady stagnation point flow whereas Lok and Pop [14] explained about unsteady separated stagnation point. On the other hand, Mahapatra et al. [15] described oblique flow. Rosali et al. [16] further investigates their study in porous medium. Zaimi and Ishak [17] used permeable stretching/shrinking sheet.

Magnetohydrodynamic (MHD) is that term in which there is existence of magnetic field on electrically conducting moving fluid. Its applications make this area special in research field for example electromagnetic pump, designing of heat exchanger, accelerator and generator. It was observed that magnetic field strength affect the viscous fluid flow as discussed by [18]–[22]. Study of stagnation point with MHD fluid flow properties done by [23] while an important part of flow which is oblique MHD flow was expanded by Lok et al. [24]. Problems related to MHD boundary layer were investigated by [25] and [26]. Bhattacharya and Krishnendu [27] performed the heat transfer and MHD flow with radiation effect in presence of heat source/sink and suction/injection. Chauhan and Agrawal [28] extended their study for shrinking sheet as well as porous substrate plate.

Casson fluid is a category of non-Newtonian fluid which has properties of shear thinning fluid that is performed by yield stress and makes the fluid flow possible otherwise fluid seems like a solid. Sauce, jelly, soup and honey are some examples of viscous fluids that are added in casson fluid whereas human blood is most appropriate example of Casson fluid which is main part of bio-medical field. Therefore, Sheikh and Abbas [29] includes Casson fluid to solve heterogeneous and homogeneous solutions.

Up to our knowledge, no study has been carried out so far to study; the flow and heat transportation on Casson fluid along with aligned MHD past a shrinking sheet. The motive of current assessment is to analyze the aligned MHD effects on Casson fluid.

2. MATERIALS AND METHODS

Steady 2D Casson fluid flow of a non-compressible, viscous, electrical conducting fluid on a shrinking sheet is considered. Aligned magnetic field are also assumed the impact on fluid flow. $u_W(x)$ and T_W are the linear velocity and uniform temperature on shrinking surface respectively (as shown in Fig. 2.1).

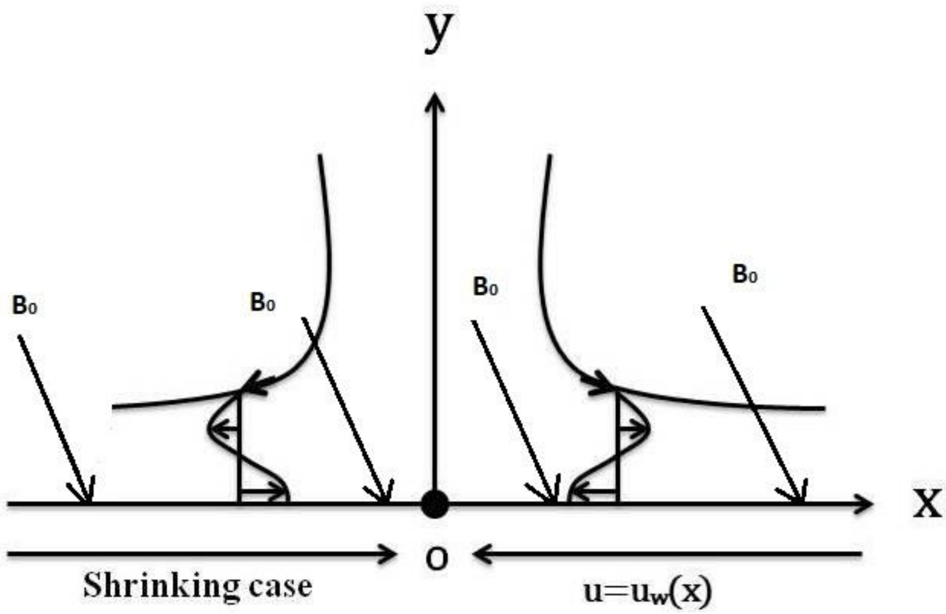


Fig. 2.1 Schematic Diagram

The generating equations of flow under the above assumptions are described as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\left(1+\frac{1}{\beta}\right)\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{\sigma B_0^2 u\sin^2 l}{\rho} \tag{2}$$

$$\rho C_P\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=K\frac{\partial^2 T}{\partial y^2} \tag{3}$$

where velocity along y (vertical axis) and x (horizontal axis) - axes are taken as v and u respectively. ν, σ, C_P, T, K and B_0 denotes the kinematic viscosity, electrical conductivity, specific heat at constant pressure, fluid temperature, thermal conductivity and magnetic field strength of the fluid respectively.

Restrictions on the boundary are describing the flow model as:

$$\left.\begin{array}{lll} y=0 & u=u_w(x)=-bx,\ v=0, & T=T_w \\ y\to\infty & u=0, & T=T_\infty \end{array}\right\} \tag{4}$$

Where b is non-negative invariable values of dimension ($time^{-1}$). The fluid having unvarying temperature T_∞ very far from the surface.

Introducing $\xi=\sqrt{\frac{b}{\nu}}x$, $\eta=\sqrt{\frac{b}{\nu}}y$ and $\psi(\xi,\eta)$ (stream function) as dimensionless variables such that $u=\frac{\partial\psi}{\partial\eta}$ and $v=-\frac{\partial\psi}{\partial\xi}$. The Boundary condition in term of stream function $\psi(\xi,\eta)$ is given by

We required solution of equation (6) from the relation $\psi=\xi f_a(\eta)$, where $f_a(\eta)$ are referred as tangential and normal parts of flow. Also, $v=-f_a(\eta)$ and $u=\xi f_a'(\eta)$.

Equation (1) is contented by given v and u and equation (2) and (4) transformed to equation (5) and (6),

$$\left(1+\frac{1}{\beta}\right)f_a'''(\eta)+f_a(\eta)f_a''(\eta)-\left(f_a'(\eta)\right)^2-M\sin^2 l f_a'(\eta)=0 \tag{5}$$

$$f_a(0)=0, f_a'(0)=-1, f_a'(\infty)=0 \tag{6}$$

Here $M=\frac{\sigma B_0^2}{b\rho}$ is the Chandershekhar number (magnetic parameter)

Dimensionless temperature $\theta(\eta)=(T-T_\infty)/(T_w-T_\infty)$. Substituting $\theta(\eta)$ in equation (3), we get

$$\theta''(\eta)+Pr\theta'(\eta)f_a(\eta)+Pr\theta(\eta)=0 \tag{7}$$

where, $Pr\ (=\mu C_P/K)$.

Corresponding boundary conditions of (4) reduces to $\theta(0)=1,\ \theta(\infty)=0$

The terms of practical importance are C_f and Nu_x are defined as

The wall shear stress τ_w is described as $C_f=\frac{\tau_w}{\rho(u_w)^2}$ then $C_f\propto f_a''(0)$ when $\tau_w=\mu\left(1+\frac{1}{\beta}\right)\frac{\partial u}{\partial y}\Big|_{y=0}$

The wall heat flux q_w is given as $Nu_x=\frac{xq_w}{K(T_w-T_\infty)}$ then $Nu_x\propto-\theta'(0)$ when $q_w=-\left(K\frac{\partial T}{\partial y}\right)\Big|_{y=0}$

3. RESULT AND DISCUSSION

Here, Runge-Kutta Fehlberg technique is considered to find out the solution of differential equations (5) and (7) with the help of shooting procedure. Velocity and dimensionless temperature of the model have been acquired for distinct entries of various fluid parameters. The value of $C_f \propto f_a''(0)$ and $Nu_x \propto -\theta'(0)$ are computed for further analysis.

Fig. 3.1 exhibits the significance of velocity profile under influence of β. Figure 3.1 expresses that the fluid velocity rise with rising entries of β. This graph elaborates that enhancement in the value of β will decline yield stress that hurdles the free movement of fluid particles and hence boundary layer thickness reduced. Figure 3.2 disclosures the behavior of β for temperature distribution. Figure 3.2 indicates that increases, fluid temperature reduced and this graph elaborates that enhancement in the value of β enhances fluid velocity that will intensify the heat transfer. We notice from the Table 3.1 that with the increase in Casson fluid parameter β, $C_f \propto f_a''(0)$ and $Nu_x \propto -\theta'(0)$ values rises.

Table-3.1 Outcomes of $f_a''(0)$ and $-\theta'(0)$ for fixed entries of $M = 10, l = \pi/3$ and $Pr = 1.1$

β	$f_a''(0)$	$-\theta'(0)$
2	2.08237	−0.85646
4	2.28068	−0.83984
8	2.40390	−0.83040

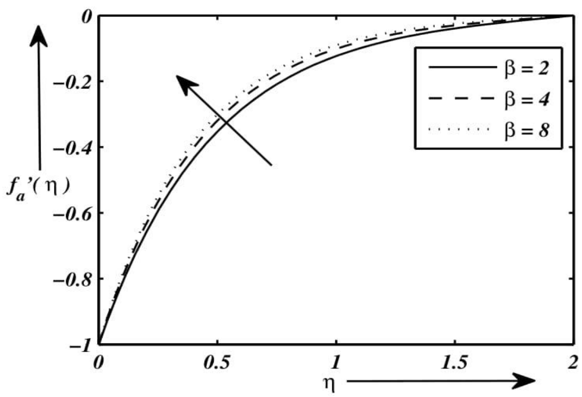


Fig. 3.1 Pattern of $f_a'(\eta)$ for distinct β with fixed entries $M = 10, l = \pi/3,\ Pr = 1.1$

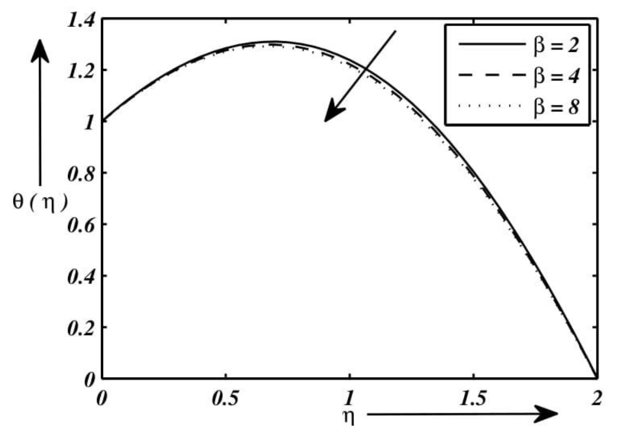
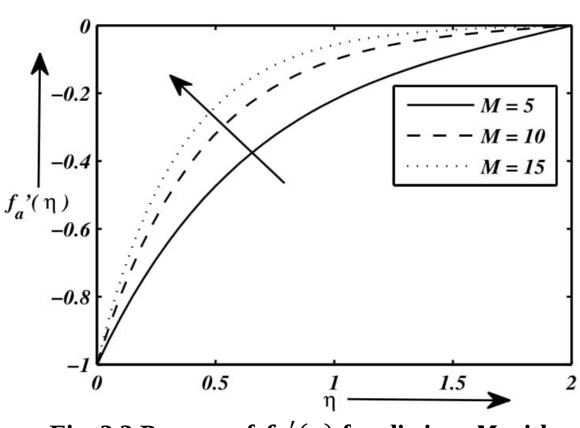


Fig. 3.2 Pattern of $\theta(\eta)$ for distinct β with fixed entries $M = 10, l = \pi/3,\ Pr = 1.1$

Fig. 3.3 manifests variation of fluid velocity against M on velocity. This figure shows that existence of magnetic parameter M resist the fluid particles to move freely and main reason behind the resistance is that M produces Lorentz force and this magnetism behavior can be adopted for controlling the fluid movement. Thus an enhancement in M causes the enhancement of velocity distribution as depicted via Figure 3.3. Figure 3.4 represents that the temperature profile declines marginally with rise in M because a slight conversion in Nu_x has been seen with rising M (shown in Table 3.2). As heat transfer rate can control by the help of magnetism. Therefore, it can conclude that in flow characteristics, M plays a substantial role. From Table 3.2, we observed that with the increase in magnetic parameter M, $C_f \propto f_a''(0)$ and $Nu_x \propto -\theta'(0)$ both are increases.



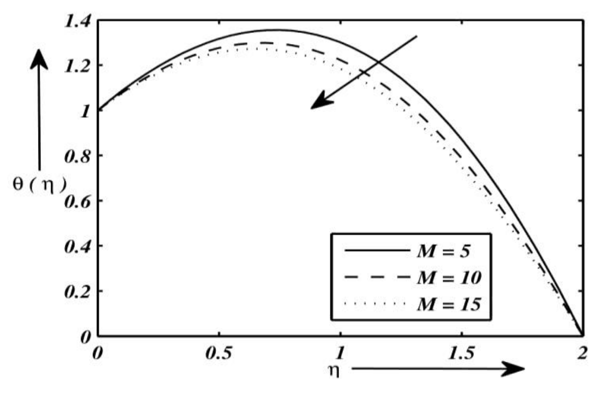


Fig. 3.3 Pattern of $f_a'(\eta)$ for distinct M with fixed entries $\beta = 4, l = \pi/3,\ Pr = 1.1$
Fig. 3.4 Pattern of $\theta(\eta)$ for distinct M with fixed entries $\beta = 4, l = \pi/3,\ Pr = 1.1$

Table-3.2 Outcomes of $f_a''(0)$ and $-\theta'(0)$ for fixed entries of $\beta = 4, l = \pi/3$ and $Pr = 1.1$

M	$f_a''(0)$	$-\theta'(0)$
5	1.48831	−0.91941
10	2.28068	−0.83984
15	2.86360	−0.80034

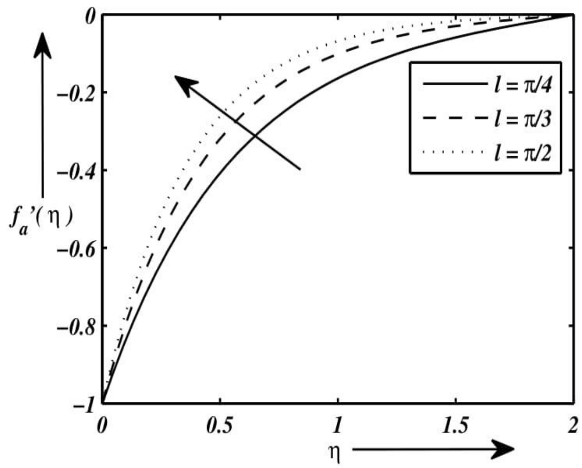


Fig. 3.5 Pattern of $f_a'(\eta)$ for distinct l with fixed entries $\beta = 4, M = 10, Pr = 1.1$

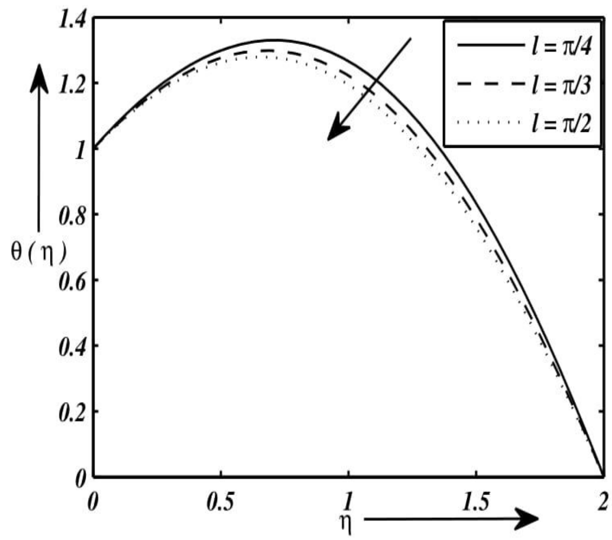


Fig. 3.6 Pattern of $\theta(\eta)$ for distinct l with fixed entries $\beta = 4, M = 10, Pr = 1.1$

Fig. 3.5 examine to analyze the impact of l on dimensionless velocity distribution. Here, increment in velocity profile has been noticed with the rising in value of l. This happened because of higher value of l that reinforces the applied magnetic field and this situation create reverse force to flow. That reverse force reduced the velocity which exhibited by Fig. 3.5. This force has capacity to reduce boundary layer thickness. In Fig. 3.6, the effect of l on temperature distribution is observed. As l increases temperature of fluid and thermal boundary thickness both reduced. From Table 3.3, we observed that with the increase in l then $C_f \propto f_a''(0)$ and $Nu_x \propto -\theta'(0)$ both are increases.

Table-3.3 Outcomes of $f_a''(0)$ and $-\theta'(0)$ for fixed entries of $\beta = 4, l = \pi/3$ and $Pr = 1.1$

l	$f_a''(0)$	$-\theta'(0)$
$\pi/4$	1.79066	−0.88472
$\pi/3$	2.28068	−0.83984
$\pi/2$	2.68336	−0.81124

4. CONCLUSION

Steady 2D Casson fluid flow of a non-compressible, viscous, orthogonally electrical conducting fluid on a shrinking sheet with has been studied. Runge-Kutta Fehlberg has been used to solve numerically the differential equations with their corresponding boundary conditions through shooting technique. The numerical results obtained in this study include the influence of aligned magnetic field and Casson parameter on temperature and velocity profiles of the flow. This analysis revealed major recommendations of the outcomes are compiled as below:

- Fluid velocity increases and momentum boundary layer thickness decreases with an increase in Casson parameter β, magnetic parameter M and aligned angle parameter l.
- Fluid temperature and thermal boundary layer thickness both decreases as we increase in Casson parameter β, magnetic parameter M and aligned angle parameter l.

These results have possible technological applications in liquid-based systems involving shrinking materials. The finding of this study may serve as to control the rate of heat transportation. A future study of this analysis can be done by considering stretching cylindrical surfaces.

5. NOMENCLATURE

R	constant	f_a	normal component of flow
β	Casson parameter	Pr	Prandtl number
x, y	cartesian coordinates	C_P	specific heat
T	dimensionaless temperature	ψ	stream function
σ	electrical conductivity	C_f	skin friction coefficient
ρ	fluid density	K	thermal counductivity
ν	kinematic viscosity	T_w	temperature at surface
q_m	local heat flux	T	temperature profile
Nu_x	local Nusselt number	T_∞	uniform ambient temperature
M	magnetic parameter (Chandershekhar number)	u, v	velocity component
B_0	magnetic field strength	τ_w	wall shear stress

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